CSE311 Microwave Engineering

LEC (08)
Impedance Matching

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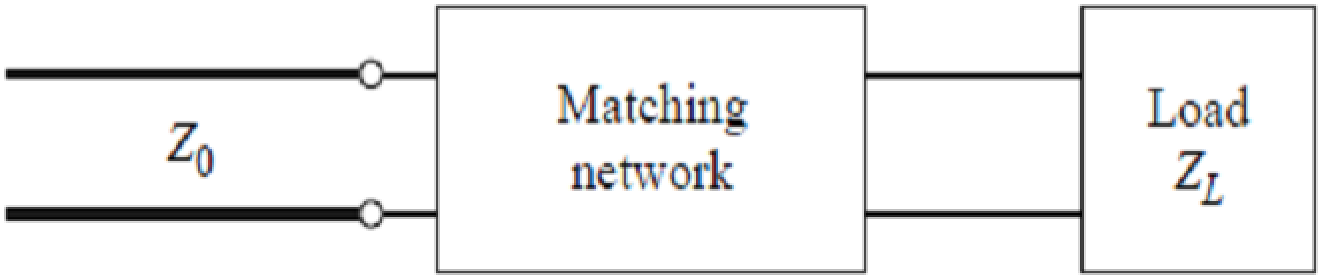
3.7 Matching Networks

- **The basic idea of impedance matching is illustrated in Figure 4.1, which shows an impedance matching network placed between a load impedance and a transmission line. The matching network is ideally lossless, to avoid unnecessary loss of power, and is usually designed so that the impedance seen looking into the matching network is Z_0.**
- **This procedure is sometimes referred to as tuning.**



Impedance matching or tuning is important for the following reasons:

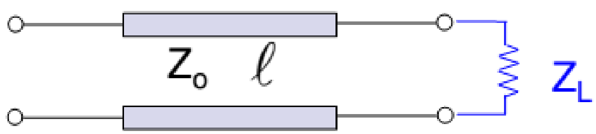
- **Maximum power is delivered when the load is matched to the line (assuming the generator is matched), and power loss in the feed line is minimized.**

3.7 Matching Networks

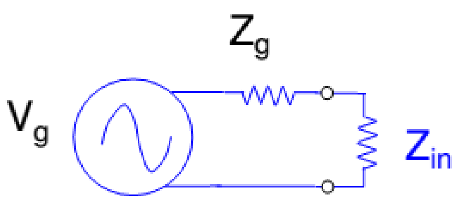
Why match?

$$\Gamma = \frac{Z - Z_o}{Z + Z_o}$$

Minimize reflection, maximize transmission

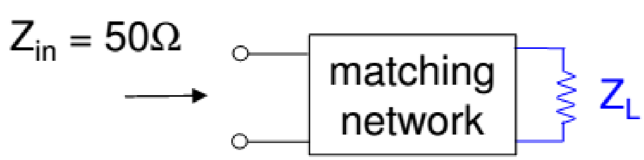


If $Z_L = Z_o$, line length is not important.



If $Z_{in} = Z_g$, max power to the load.

Goal: to design matching networks so all Z's are the same = 50Ω



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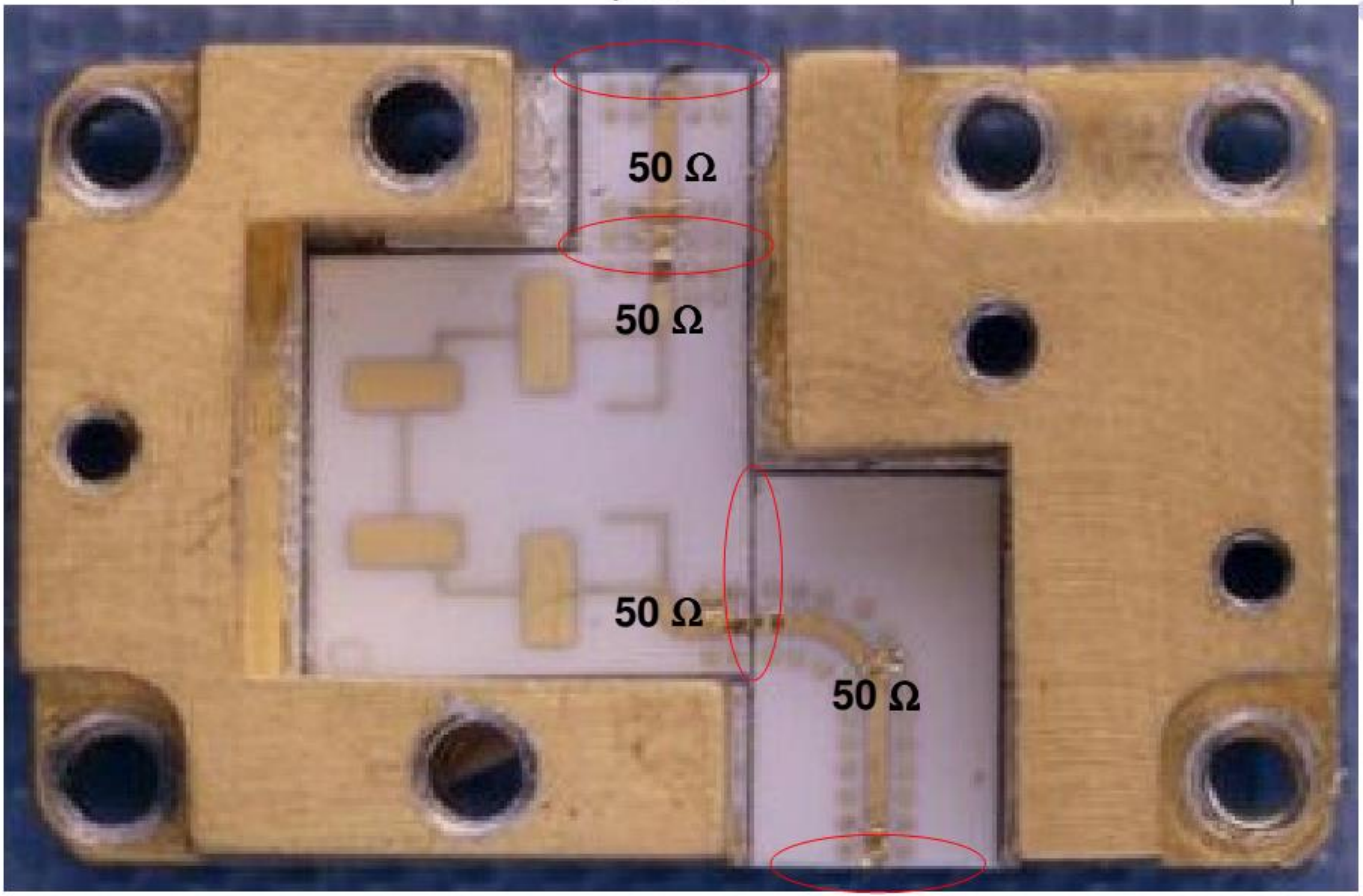
Technician tuning even for 50Ω designs

input 50 Ω

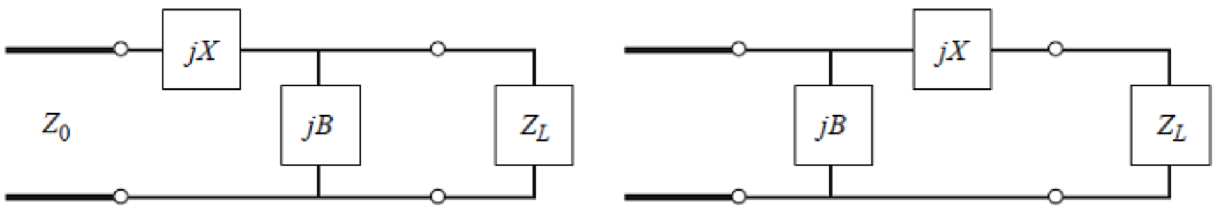


output 50 Ω

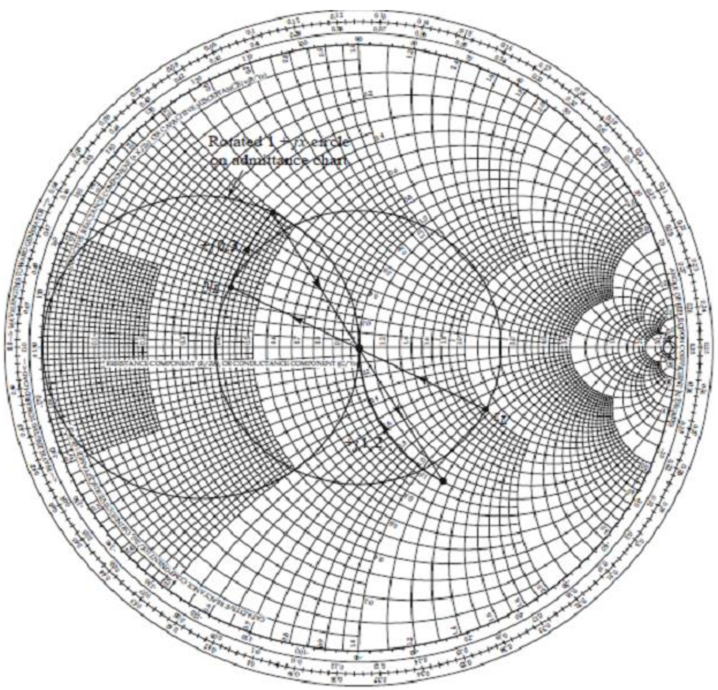
3.7 Matching Networks

Matching with Lumped Elements (L – Networks)

- **Probably the simplest type of matching network is the L-section, which uses two reactive elements to match an arbitrary load impedance to a transmission line.**
- **There are two possible configurations for this network, as shown in Figure 4.2.**

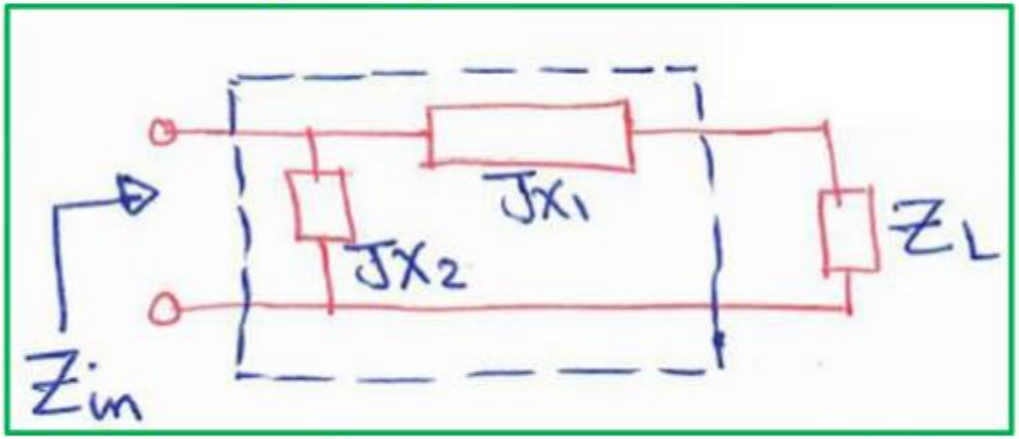


(a)



3.7 Matching Networks

- **Analytic Solution for $Z_L < Z_0$**



- **Rearranging and separating into real and imaginary parts gives:**

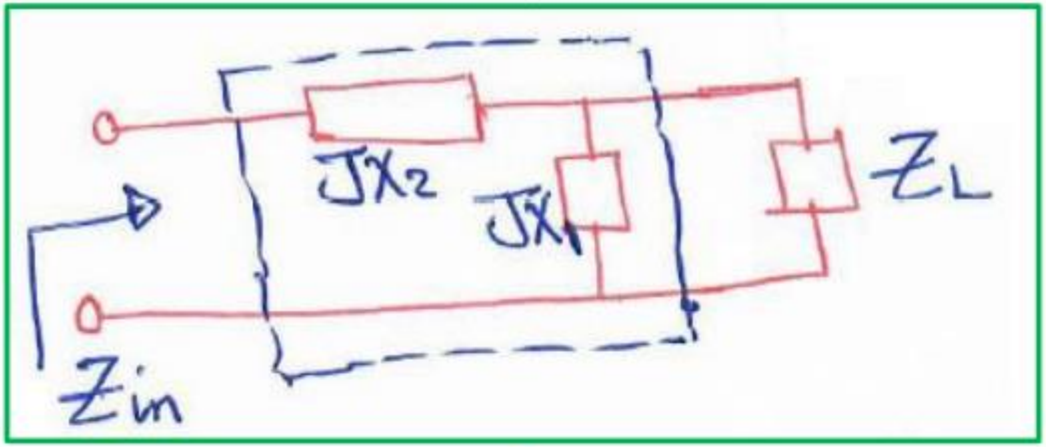
$$Z_{in} = (Z_L + jX_1) \,//\, jX_2 = Z_0$$

$$\frac{1}{Z_L + jX_1} + \frac{1}{jX_2} = \frac{1}{Z_0}$$

$$\frac{1}{Z_L + jX_1} = \underbrace{\boxed{\frac{1}{Z_0}}}_{\text{real}} + \underbrace{\boxed{\frac{j}{X_2}}}_{\text{imaginary}}$$

3.7 Matching Networks

- **Analytic Solution for $Z_L > Z_0$**



- **Rearranging and separating into real and imaginary parts gives:**

$$Z_{in} = jX_2 + \frac{jX_1 * Z_L}{jX_1 + Z_L} = Z_0$$

$$\frac{jX_1 * Z_L}{jX_1 + Z_L} = \underbrace{\boxed{Z_0}}_{\text{real}} - \underbrace{\boxed{jX_2}}_{\text{imaginary}}$$

3.7 Matching Networks

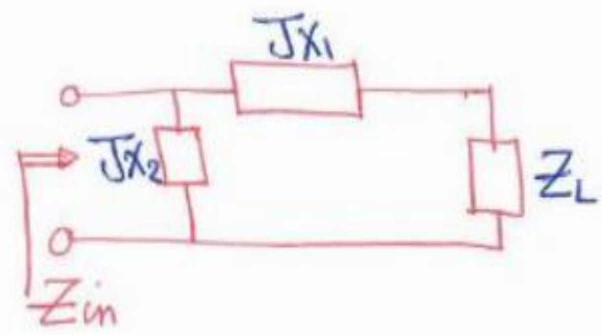
Example 1

Design an L-section matching Circuit for $Z_L = 60 + J40$ for $Z_o = 100\,\Omega$ and $f = 1\,MHz$.

Solution

$$Z_L = 60 + J40\,\Omega \quad \text{and} \quad Z_o = 100\,\Omega$$

$$\therefore \underline{\underline{Z_L < Z_o}}$$



$$\frac{1}{Z_L + JX_1} = \frac{1}{60 + J40 + JX_1}$$

$$\frac{1}{Z_L + JX_1} = \frac{1}{60 + J(40 + X_1)} * \frac{60 - J(40 + X_1)}{60 - J(40 + X_1)}$$

3.7 Matching Networks

Example 1

$$\frac{1}{Z_L + jX_1} = \frac{60 - j(40 + X_1)}{(60)^2 + (40 + X_1)^2}$$

For real part

$$\frac{60}{(60)^2 + (40 + X_1)^2} = \frac{1}{Z_0} = \frac{1}{100}$$

$$X_1^2 + 80X_1 + (60)^2 + (40)^2 - 60 * 100 = 0$$

$$\boxed{X_1 = 8{,}99} \quad \text{or} \quad \boxed{X_1 = -88{,}99}$$

For imaginary part

$$\frac{j}{X_2} = \frac{-j(40 + X_1)}{(60)^2 + (40 + X_1)^2} \quad \boxed{X_2 = -122{,}5} \quad \text{or} \quad \boxed{X_2 = 122{,}5}$$

3.7 Matching Networks

Example 1

N.B

$$jX = \left[\begin{array}{l} j\omega L \\ \frac{-j}{\omega C} \end{array}\right.$$

First Solution

$X_1 = 8,99$ and $X_2 = -122,5$

$jX_1 = j\omega L$

$$L = \frac{8,99}{2\pi f} = \frac{8,99}{2\pi * 10^6}$$

$$L = 1,43\ \mu H$$

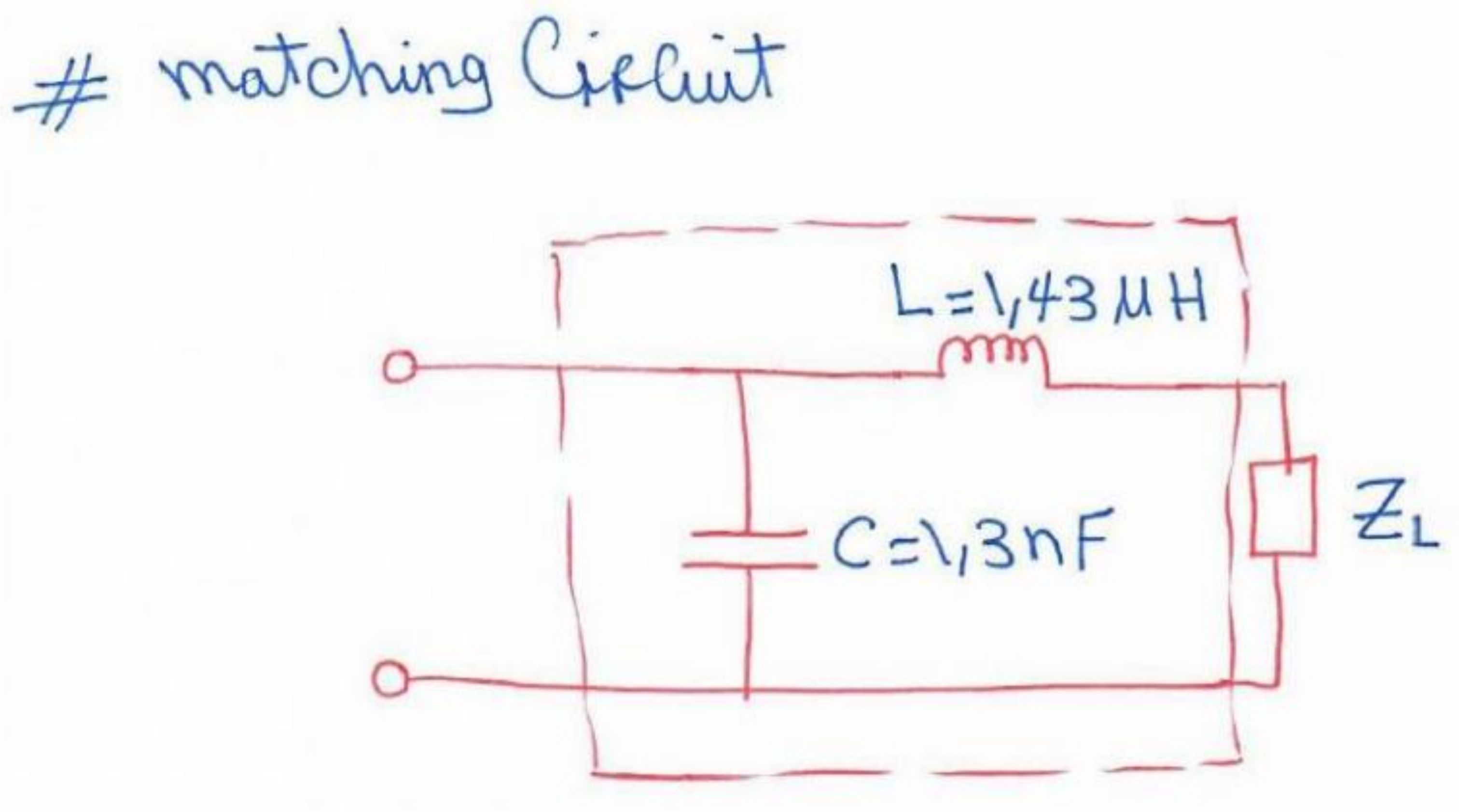
$jX_2 = \frac{-j}{\omega C}$

$$122,5 = \frac{1}{2\pi * 10^6 * C}$$

$$C = 1,3\ nF$$

3.7 Matching Networks

Example 1



3.7 Matching Networks

Example 1

Second Solution

$X_1 = -88{,}99$ and $X_2 = 122{,}5$

$jX_1 = \frac{-j}{\omega C}$

$jX_2 = j\omega L$

$88{,}99 = \frac{1}{2\pi * 10^6 * C}$

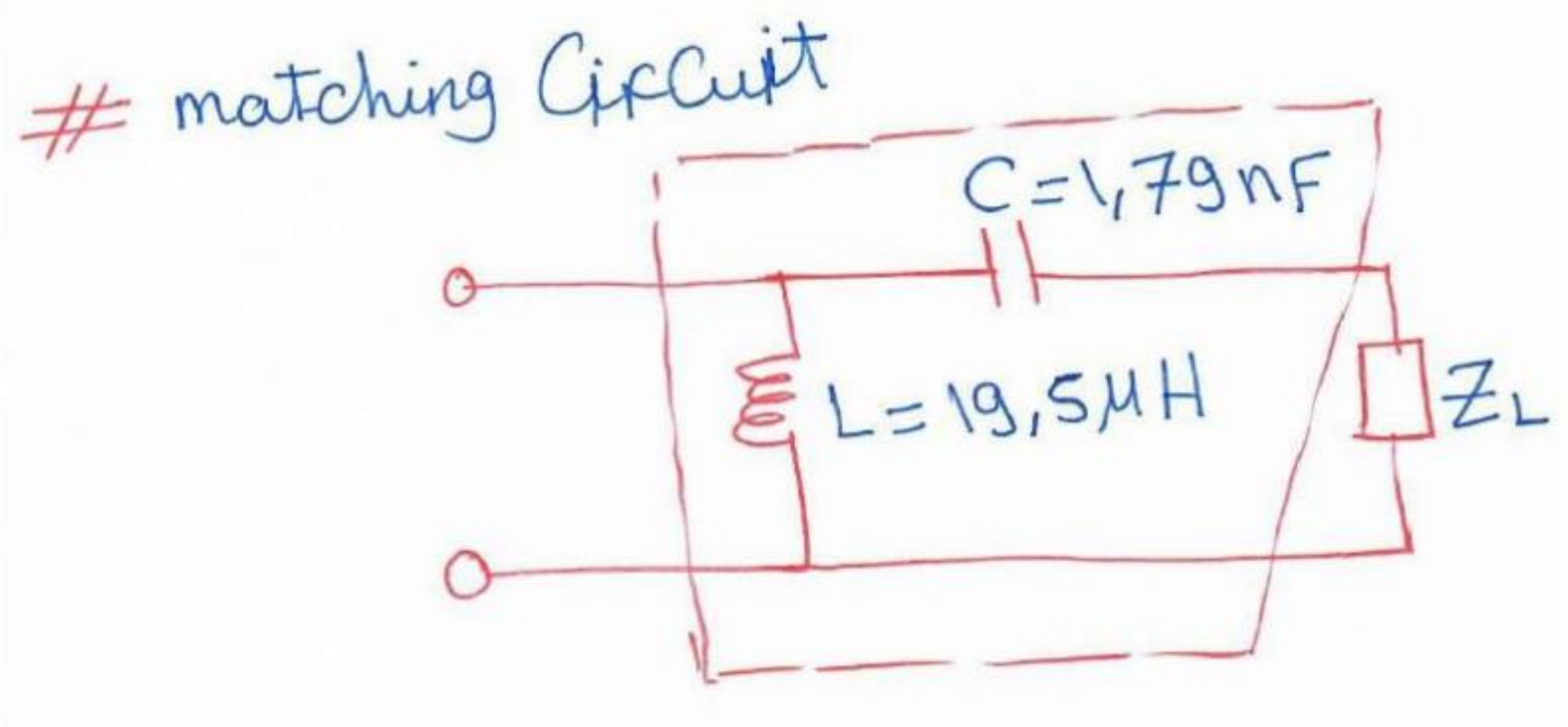
$L = \frac{122{,}5}{2\pi * 10^6}$

$C = 1{,}79\,nF$

$L = 19{,}5\,\mu H$

3.7 Matching Networks

Example 1



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Example 2 Design an L-Section matching Circuit for $Z_L = 150 + J40$ with $Z_o = 100\,\Omega$ and $f = 1\,MHz$

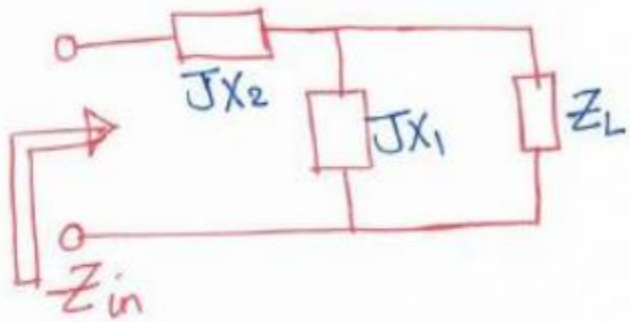
Solution

$Z_L = 150 + J40\,\Omega$ and $Z_o = 100\,\Omega$

$$\therefore \underline{\underline{Z_L > Z_o}}$$

$$Z_{in} = JX_2 + \frac{JX_1 * Z_L}{Z_L + JX_1}$$

$$Z_{in} = Z_o$$



$$\frac{JX_1 * Z_L}{JX_1 + Z_L} = Z_o - JX_2$$

3.7 Matching Networks

Example 2

$$\therefore \quad \frac{JX_1(150+J40)}{150+J40+JX_1} * \frac{150-J(40+X_1)}{150-(40+X_1)} =$$

$$= \frac{JX_1\left[(150)^2 + J6000 - J150(40+X_1) + 40(40+X_1)\right]}{(150)^2+(40+X_1)^2}$$

$$= \frac{150\,X_1^2 + J\left[X_1 * (150)^2 + 1600X_1 + 40X_1^2\right]}{(150)^2+(40+X_1)^2}$$

3.7 Matching Networks

Example 2

For real part

$$\frac{150X_1^2}{(150)^2+(40+X_1)^2} = Z_o = 100$$

$$100\left[(150)^2+(40+X_1)^2\right] = 150X_1^2$$

$$2\left[(150)^2+(40)^2+80X_1+X_1^2\right] = 3X_1^2$$

$$X_1^2-160X_1-2\left[(150)^2+(40)^2\right] = 0$$

$$\boxed{X_1 = 313{,}67} \quad \underline{\text{or}} \quad \boxed{X_1 = -153{,}67}$$

For imaginary part

$$-JX_2 = \frac{(150)^2X_1+1600X_1+40X_1^2}{(150)^2+(40+X_1)^2}$$

$$\boxed{X_2 = 77{,}89} \quad \underline{\text{or}} \quad \boxed{X_2 = -77{,}89}$$

3.7 Matching Networks

Example 2

First Solution

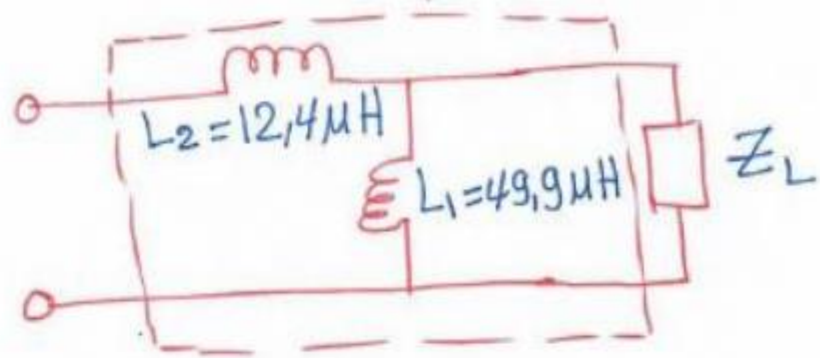
$X_1 = 313{,}67$ and $X_2 = 77{,}89$

$$L_1 = \frac{313{,}67}{2\pi * f}$$

$$L_2 = \frac{77{,}89}{2\pi * f}$$

$L_1 = 49{,}9\,\mu H$

$L_2 = 12{,}4\,\mu H$



3.7 Matching Networks

Example 2

Second Solution

$X_1 = -153{,}67$ and $X_2 = -77{,}89$

$$C_1 = \frac{1}{153{,}67 * 2\pi * f}$$

$$C_2 = \frac{1}{77{,}89 * 2\pi * f}$$

$C_1 = 1{,}036\,nF$

$C_2 = 2{,}044\,nF$